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Coulomb Forces in the Three-Body Problem with Application to Direct Nuclear Reactions

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Faddeev equations are considered in the case of three charged particles interacting with both separable nuclear two-body interactions and also including Coulomb forces. Modified Faddeev equations with Coulomb Green's functions are introduced. The three-body amplitudes are given into pure Coulomb and distorted-Coulomb amplitudes. Introducing a decomposition in the angular momentum states, a set of three-body integral equations is obtained. The effect of pure coulomb amplitudes is studied in direct nuclear reactions and found to give a large contribution to the cross sections. The three-body integral equations obtained are applied for direct nuclear reactions. The angular distributions for $^{12}C(^{6}Li, d)^{16}O$, $^{16}O(^{6}Li, d)^{20}Ne$, and $^{12}C(^{6}Li, \alpha)^{14}N$ transfer reactions are calculated as well as for the ^{6}Li elastic scattering on ^{12}C. From the good agreement between the theoretically calculated and experimental data, better spectroscopic factors are extracted. The effect of including Coulomb forces in the three-body problem is found to improve the results by about 16.26%.

1. INTRODUCTION

One of the most interesting three-body problems is that of Coulomb forces. Separable potentials have been shown to be useful in solving the three-body problem. The Coulomb force is of quite different nature. The inclusion of Coulomb forces in the three-body problem has been considered by several authors (Schulman, 1967; Noble, 1967; Alt et al. 1967; Nutt, 1968; Hamza and Edwards, 1969; Osman, 1971a). In all these approaches for a system of three charged particles, it is necessary to know the two-body Coulomb T matrix off the energy shell. Schulman (1967) suggests approximating the Coulomb Green's functions in momentum space. Another

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suggestion is the improved version of the Schulman approximation based on the Yamaguchi potential. In all cases, the Faddeev kernels (Faddeev, 1960; 1961; 1962) still contain the two-body Coulomb T matrix. Including Coulomb forces in the three-body system introduces modified Faddeev equations and Coulomb Green's functions (Osman, 1971a). This approach is applied for different three-body problems (Osman, 1971a; 1977; 1978a–c; 1979).

In the present work, the Coulomb forces are included in the three-body problem to be applied for direct nuclear reactions. We consider a system of three interacting, charged particles. The nuclear two-body interactions are taken as nonlocal separable potentials. The two-body Coulomb forces are included. Coulomb Green's functions are defined by approximating the Coulomb wave functions in momentum space. This is done keeping in mind that (Noble, 1967) the Coulomb potentials act for distances which are much larger than the ranges of the other interactions involved in the problem. We follow here the Faddeev–Lovelace (Faddeev, 1965; Lovelace, 1964) formalism. Modified Faddeev equations are obtained which form a set of coupled integral equations. The obtained equations are manageable and are suitable for computation.

In the present work, we consider the direct transfer nuclear reactions $^{12}C(^{6}Li, d)^{16}O$, $^{16}O(^{6}Li, d)^{20}Ne$, and $^{12}C(^{6}Li, \alpha)^{14}N$. In these reactions, the projectile nucleus ^{6}Li is taken as a cluster composition of a deuteron and an alpha particle. Thus, we have in the initial channel a three-body problem of the three charged interacting particles, the deuteron, the alpha particle, and the target nucleus. Two of these particles are bound (the deuteron and the alpha particles, forming the projectile ^{6}Li nucleus), and the third particle is free (the target nucleus, ^{12}C for the first and third reactions and ^{16}O for the second reaction). In the final channel, we also have a three-body problem of three charged interacting particles, two of which are bound (the transferred particle with the target nucleus, forming the residual nucleus), and the third particle is free (the outgoing particle). In the final channels of the three reactions considered, the transferred alpha particle is bound with the ^{12}C nucleus forming ^{16}O in the first reaction and the alpha particle is bound to the ^{16}O target forming ^{20}Ne in the second reaction while in the third reaction, the transferred deuteron is bound with the ^{12}C target forming the ^{14}N nucleus. Numerical calculations are performed for the integral equations obtained, including the Coulomb forces between the interacting particles. Differential cross sections for these direct transfer nuclear reactions are calculated. Also, the ^{6}Li elastic scattering on ^{12}C is considered. Angular distributions are compared with the experimental measurements. From the fitting of angular distributions between the theoretical and experimental data, the spectroscopic factors are extracted.

In Section 2, we introduce the three-body integral equations including Coulomb forces. Calculations and results are presented in Section 3. Section 4 is devoted to discussion and conclusions.

2. THREE-BODY EQUATIONS INCLUDING COULOMB FORCES

Considering the system of the charged three particles to be labeled by 1, 2, and 3 with masses m_1, m_2, and m_3 and with momenta $\mathbf{k}_1$, $\mathbf{k}_2$, and $\mathbf{k}_3$ in the center of mass of the three-body system. Following the Faddeev–Lovelace formalism (Lovelace, 1964), $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$ are the center of mass momenta of the (2,3), (3,1), and (1,2) pairs and $\mathbf{q}_1$, $\mathbf{q}_2$, and $\mathbf{q}_3$ are the momenta of particle 1 relative to the subsystem (2,3), of particle 2 relative to the (3,1) subsystem, and of particle 3 relative to the (1,2) subsystem and given by

$$\mathbf{p}_1 = (m_3\mathbf{k}_2 - m_2\mathbf{k}_3)/[2m_2m_3(m_2+m_3)]^{1/2} \tag{1}$$

and

$$\mathbf{q}_1 = [m_1(\mathbf{k}_2+\mathbf{k}_3) - (m_2+m_3)\mathbf{k}_1]/[2m_1(m_2+m_3)(m_1+m_2+m_3)]^{1/2} \tag{2}$$

Then the system has a kinetic energy in the center-of-mass system as

$$H_0 = p_1^2 + q_1^2 \tag{3}$$

with similar forms for $\mathbf{p}_2$, $\mathbf{q}_2$ and $\mathbf{p}_3, \mathbf{q}_3$ by cyclic permutations of 1, 2, and 3.

The total Hamiltonian of the three particle system is given by

$$H = H_0 + U + V \tag{4}$$

where U is the sum of the Coulomb potentials and V is the sum of the short-range nuclear interactions. In the present work, we neglect the three-body Coulomb potential, and then U is the sum of the two-body Coulomb potentials.

Now, introducing the two-body Coulomb Green's functions as

$$G_{ij}^C(Z) = (H_0 + U_{ij} - Z)^{-1} \tag{5}$$

Then we get

$$\langle \mathbf{p}_i, \mathbf{q}_i | G_{jk}^C(Z) | \mathbf{p}_i', \mathbf{q}_i' \rangle = \delta(\mathbf{q}_i' - \mathbf{q}_i) \langle \mathbf{p}_i | G_{jk}^C(Z - q_i^2) | \mathbf{p}_i' \rangle \tag{6}$$

which can be written as

$$\langle \mathbf{p}_i, \mathbf{q}_i | G_{ij}^C(Z) | \mathbf{p}_i', \mathbf{q}_i' \rangle = \delta(\mathbf{q}_i' - \mathbf{q}_i) \int \frac{d^3k \langle \mathbf{p}_i | \psi_k \rangle \langle \psi_k | \mathbf{p}_i' \rangle}{q_i^2 + k^2 - Z} \tag{7}$$

The Coulomb wave function $\langle \mathbf{p} | \psi_k \rangle$ in momentum space will be peaked when $\mathbf{p}_i$ coincides in direction and magnitude with the integral variable in equation (6), and then the Coulomb Green's function can be transformed to coordinate system as (Schulman, 1967)

$$\begin{aligned} \int \psi_k(\mathbf{p}) f(\mathbf{p}) d^3p &\cong f(\mathbf{k}) \int \psi_k(\mathbf{p}) d^3p \\ &= f(\mathbf{k}) \left[\Psi_k^C(\mathbf{r}) \right]_{r=0} \end{aligned} \tag{8}$$

where $[\Psi_k^C(\mathbf{r})]_{r=0}$ is the Coulomb wave function in configuration space and is given by

$$\left[\Psi_k^C(\mathbf{r}) \right]_{r=0} = \left[\frac{2\pi\eta_{jk}}{\exp(2\pi\eta_{jk}) - 1} \right]^{1/2} \tag{9}$$

$\eta_{jk} = \mu_{jk} Z_j Z_k e^2/k$ is the Coulomb parameter, Z_j and Z_k are the charge numbers of the particles j and k, and μ_{jk} is the reduced mass of the (j, k) subsystem.

Using a Yamaguchi (1954a, b) form for the wave function with a form factor given by

$$\langle i | \mathbf{p}_i \rangle = g_i(p_i) = N_i \frac{1}{p_i^2 + \beta_i^2} \tag{10}$$

Then, the matrix element for the Coulomb Green's functions is given by

$$\begin{aligned} \langle \mathbf{p}_i, \mathbf{q}_i | G_{jk}^C(Z) | \mathbf{p}_i', \mathbf{q}_i' \rangle &\approx \delta(\mathbf{p}_i - \mathbf{p}_i') \delta(\mathbf{q}_i - \mathbf{q}_i') \\ &\times \frac{\left| \left[\Psi_{p_i}^C(\mathbf{r}) \right]_{r=0} \exp\left[2\eta_{jk} \tan^{-1}(q_i/\beta_i) \right] \right|^2}{p_i^2 + q_i^2 - Z} \end{aligned} \tag{11}$$

With this definition for the Coulomb Green's functions, we can proceed to obtain the two-body amplitudes containing both of the nuclear and Coulomb potentials. If the short-range nuclear potentials are taken to have a Yamaguchi type (1954a, b), as a nonlocal separable potential, we can write

$$V_i = \lambda_i |i\rangle\langle i| \tag{12}$$

V_i is the two-particle short-range nuclear potential between the particles j and $k(V_{jk})$. Then the two-particle amplitudes including both of the nuclear and Coulomb potentials are defined in the three-particle Hilbert space as

$$T_i(Z) = V_i + V_i G_{jk}^C(Z) T_i(Z) \tag{13}$$

which, with the separable form for the potentials V_i given by equation (12), have solutions as

$$\langle \mathbf{q}_i | T_i(Z) | \mathbf{q}_i' \rangle = |i\rangle\langle \mathbf{q}_i | \big[\langle i | G_{jk}^C(Z) \\ \times G_0(q_i^2 - E_i) |i\rangle \times (Z - q_i^2 - E_i) \big]^{-1} |\mathbf{q}_i'\rangle\langle i| \tag{14}$$

where

$$G_0(Z) = (H_0 - Z)^{-1} \tag{15}$$

and i, j, $k = 1,2,3$ in cyclic permutation. Then the corresponding three-body equations can be given as a set of coupled integral equations. In obtaining these equations, we follow the Faddeev–Lovelace formalism.

In addition to the pure Coulomb amplitudes, other three-body amplitudes must be added. These amplitudes are the on-the-energy-shell amplitudes. Following also the Faddeev–Lovelace formalism and after a lengthy mathematical work which has no place here, we get

$$\begin{aligned} f_{i\nu}(Z) = {} & -(1-\delta_{i\nu})\langle i | G_{jk}^C(Z) | \nu \rangle \\ & - \sum_{\mu} f_{i\mu}(Z) \big[\lambda_\mu^{-1} + \langle \mu | G_{ij}^C(Z) | \mu \rangle \big]^{-1} \\ & \times (1-\delta_{\mu\nu}) \langle \mu | G_{jk}^C(Z) | \nu \rangle \end{aligned} \tag{16}$$

Let us introduce the notations

$$B_{i\nu}(Z) = (1-\delta_{i\nu}) \langle i | G_{jk}^C(Z) | \nu \rangle \tag{17}$$

and

$$Y_\mu(Z) = \left[\lambda_\mu^{-1} + \langle \mu | G_{jk}^C(Z) | \mu \rangle\right]^{-1} \tag{18}$$

Thus equation (16) is given as

$$f_{i\nu}(Z) = -B_{i\nu}(Z) - \sum_\mu f_{i\mu}(Z) Y_\mu(Z) B_{\mu\nu}(Z) \tag{19}$$

For the pure Coulomb contributions, we neglect the three-body pure Coulomb forces and we only consider contributions from two-body Coulomb forces. In addition to the pure Coulomb contribution, we must add the three-body amplitudes given by equation (19). The expression given by equation (19) is an integral equation with three-dimensional integral. To simplify it, we use partial wave analysis which helps in eliminating two of the variables of integration. The partial wave analysis is introduced as

$$\langle \mathbf{q} | f_{i\nu}(Z) | \mathbf{q}' \rangle = \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) f_{i\nu}^l(q, q'; Z) \tag{20}$$

where θ is the angle between q and q'. Also we have for $B_{i\nu}(Z)$ the partial wave expansion

$$\langle \mathbf{q} | B_{i\nu}(Z) | \mathbf{q}' \rangle = \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) B_{i\nu}^l(q, q'; Z) \tag{21}$$

Using equations (20) and (21), then equation (19) is reduced in partial waves to a form given by

$$f_{i\nu}^l(q, q'; Z) = -B_{i\nu}^l(q, q'; Z) - 4\pi \sum_\mu \int q''^2 dq'' f_{i\mu}^l(q, q''; Z) \times Y_\mu(q''; Z) B_{\mu\nu}^l(q', q''; Z) \tag{22}$$

The expression given by equation (22) is a one-dimensional integral equation.

For direct transfer nuclear reactions, the differential cross section is given by

$$\frac{d\sigma_{if}}{d\Omega} = \frac{\overset{*}{m}_i \overset{*}{m}_f}{(2\pi\hbar^2)^2} \frac{k_f}{k_i} \frac{(2I_R+1)}{(2I_T+1)} \sum_{\substack{\mu_I \mu_T \\ \mu_c \mu_R}} |\langle I_I \mu_I, I_T \mu_T; \mathbf{q} | f_{if} | \mathbf{q}'; I_c \mu_c, I_R \mu_R \rangle|^2 \tag{23}$$

which represent a transition from the initial channel i (of incident projectile I and target nucleus T), to a final channel f (of outgoing particle C and residual nucleus R). $\overset{*}{m}_i$ and $\overset{*}{m}_f$ are the reduced masses of the initial and final channels, respectively. I_i and μ_i are the spin and its projection of the particle i. We neglect here the isospin since the Coulomb force breaks isospin symmetry.

3. CALCULATIONS AND RESULTS

In the present work, we are interested in calculating the effect of including Coulomb forces in the three-body problem. The presently obtained expressions are applied to direct transfer nuclear reactions. An interesting example to show this effect is the ^{6}Li induced reactions with alpha particle transfer or with deuteron transfer. The ^{6}Li projectile is considered as a cluster structure (Wildermuth & McClure, 1966; Rotter, 1966) of bound state of an alpha particle and a deuteron with binding energy of 1.47 MeV. The parameters of the two-particle interactions for the separable potentials given by equation (10) are determined from the two-body data. N_i are determined by normalizing the corresponding wave function giving

$$N_i^2 = \beta_i \varepsilon_i^{1/2}\left(\varepsilon_i^{1/2} + \beta_i\right)^3/\pi^2 \tag{24}$$

where ε_i is the binding energy of the i pair particles. Then β_i and N_i are determined independently to fit the i pair (the bound state of the j and k particles), data of binding energy ε_i and scattering length.

Performing numerical integrations, the bound-state poles are obtained by defining the λ_i's at the corresponding binding energy for each pair of particles. We follow in the present calculations for performing the numerical integrations, the Kopal (1955) method. This method is used in computing the different λ_i's and also in the partial wave analysis by solving the integrals given by equation (22). These integrals are replaced by a 36-point mesh.

Numerical calculations for ^{6}Li-induced reactions with direct transfer of alpha particle or a deuteron are performed. The differential cross sections for ^{6}Li stripping reactions are calculated using the obtained integral equations. A comparison of the angular distributions with the experimental data of Becchetti et al. (1978) for the reaction ^{12}C(^{6}Li, d)^{16}O, of Anantaraman et al. (1979) for the reaction ^{16}O(^{6}Li, d)^{20}Ne, and of White et al. (1973, 1975)

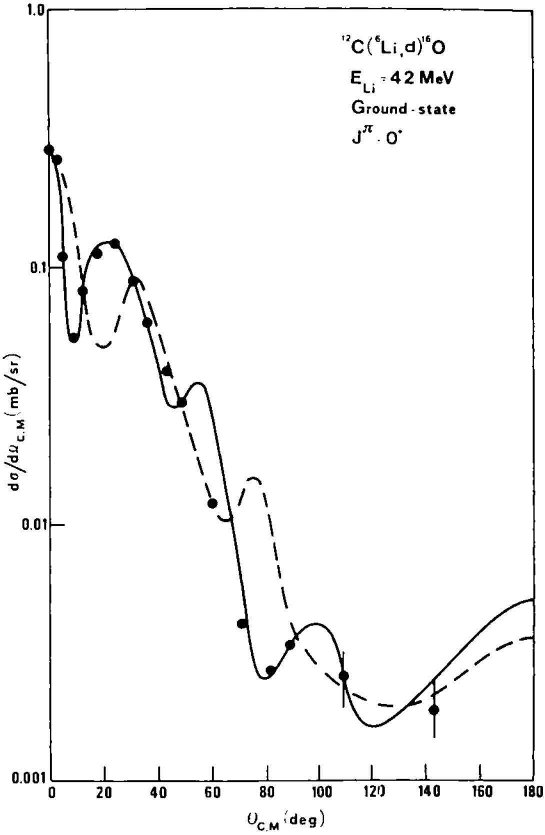

Fig. 1. The angular distributions of the ^{6}Li stripping reaction $^{12}C(^6Li, d)^{16}O$ at ^{6}Li incident energy of 42 MeV leaving the ^{16}O nucleus in its ground state. The solid curve is our present calculations. The dashed curve is calculated according to our previous model introduced in Osman (1971b, 1972). The experimental data are taken from Becchetti et al. (1978).

for the reaction $^{12}C(^6Li, \alpha)^{14}N$ are introduced in Figures 1–3, respectively. The alpha particle transfer reaction $^{12}C(^6Li, d)^{16}O$ shown in Figure 1 is performed at ^{6}Li incident energy of 42 MeV. The $^{16}O(^6Li, d)^{20}Ne$ reaction with alpha particle transfer shown in Figure 2 is performed at ^{6}Li incident energy of 32 MeV. For the deuteron transfer reaction $^{12}C(^6Li, \alpha)^{14}N$ shown in Figure 3, the ^{6}Li projectile energy is 33 MeV. To compare the present results including Coulomb forces with calculations which do not contain the Coulomb forces, numerical calculations are done for three-body problem of ^{6}Li induced reactions using a model introduced by us (Osman, 1971b; 1972). The calculations due to the present work are shown by solid curves

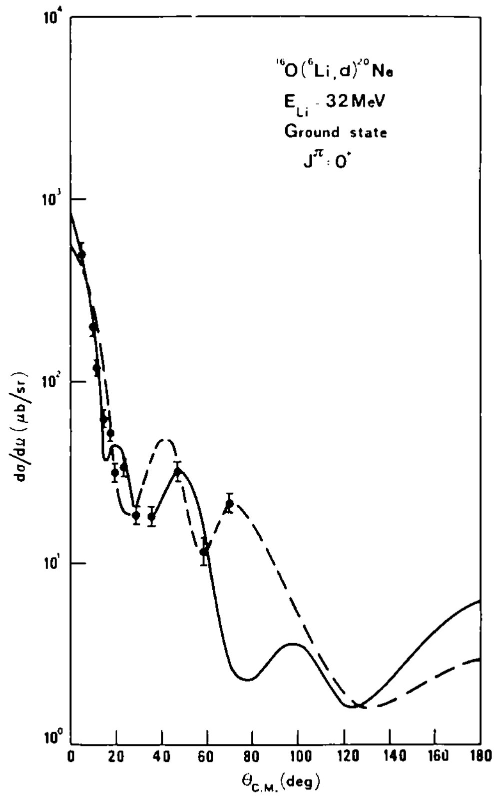

Fig. 2. The angular distributions of the ^{6}Li stripping reaction $^{16}O(^6Li, d)^{20}Ne$ at ^{6}Li incident energy of 32 MeV leaving the ^{20}Ne nucleus in its ground state. The solid curve is our present calculations. The dashed curve is calculated according to our previous model introduced in Osman (1971b, 1972). The experimental data are taken from Anantaraman et al. (1979).

on Figures 1–3, while calculations due to our previous model (Osman, 1971b; 1972) are shown by dashed curves. Also, the elastic scattering of ^{6}Li particle on ^{12}C target at ^{6}Li incident energy of 42 MeV is shown in Figure 4. The agreement between the present theoretically calculated values and the experimental measurements are good as shown in Figures 1–4. Spectroscopic factors are extracted from both calculations for the purpose of comparison, and are listed in Table I. From the values of the spectroscopic factors, we see that the effect of including the Coulomb forces in the three-body problem in the case of ^{6}Li-induced reactions is improving the results by a percentage of between 11.7254% and 24.1560%.

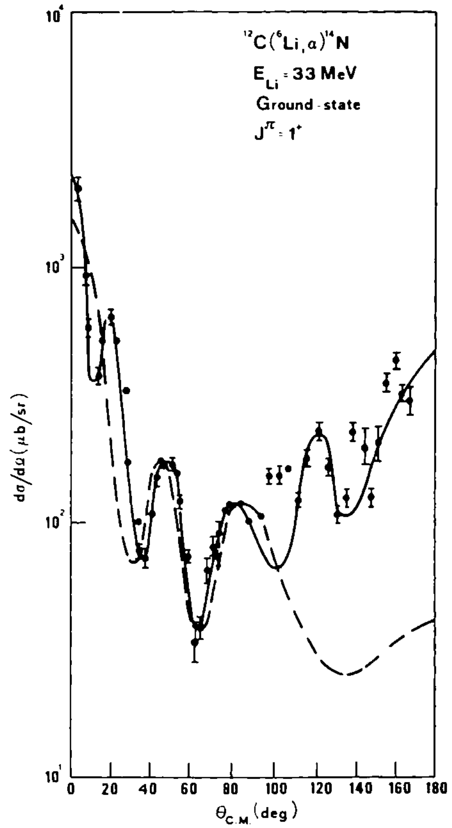

Fig. 3. The angular distributions of the ^{6}Li stripping reaction $^{12}C(^6Li, \alpha)^{14}N$ at ^{6}Li inciden energy of 33 MeV leaving the ^{14}N nucleus in its ground state. The solid curve is our presen calculations. The dashed curve is calculated according to our previous model introduced ii Osman (1971b, 1972). The experimental data are taken from White (1973) and White et al (1975).

4. DISCUSSION AND CONCLUSIONS

In the present work we solved the three-body problem of three interac ing charged particles. The Coulomb forces are included in the three-bod equations. The obtained three-body amplitudes are given into pure Coulom amplitudes and distorted-Coulomb amplitudes. The obtained equations a a set of coupled integral equations. These integral equations are manageab for computational calculations. The obtained integral equations are numei cally calculated and applied to direct transfer nuclear reactions. Strippii

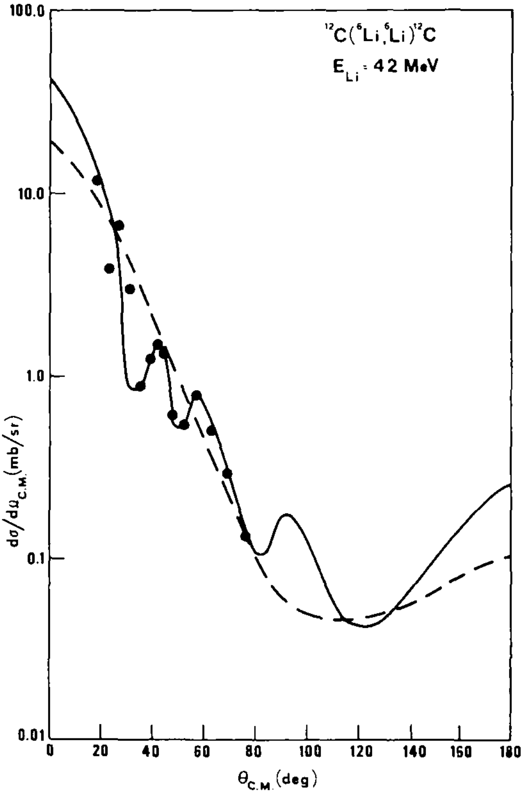

Fig. 4. The angular distributions of the ^6Li elastic scattering reaction on ^{12}C at ^6Li incident energy of 42 MeV. The solid curve is our present calculations. The dashed curve is calculated according to our previous model introduced in Osman (1971b, 1972). The experimental data are taken from Becchetti et al. (1978).

TABLE I. Extracted Spectroscopic Factors

Reaction	Incident energy (MeV)	Excitation energy (MeV)	J^{π}	Spectroscopic factors: Present work	Spectroscopic factors: Our previous model[a]	$S_{present}/S_{previous}$ (%)
$^{12}C(^6Li, d)^{16}O$	42	0.0	0^+	0.8916	0.7834	11.7254
$^{16}O(^6Li, d)^{20}Ne$	32	0.0	0^+	0.8684	0.7632	12.8953
$^{12}C(^6Li, \alpha)^{14}N$	33	0.0	1^+	0.9428	0.7396	24.1560

[a]See Osman (1971b, 1972).

reactions with ^{6}Li projectiles are shown in Figures 1–3. The agreement between the theoretical and experimental angular distributions is good. The calculated differential cross sections give the typical stripping pattern, showing an increase in the forward and backward angles with some peaks in between. For the ^{6}Li elastic scattering calculations shown in Figure 4, the absolute values of the predicted cross sections are in qualitative agreement with the experimental values. The backward peak appeared in Figures 1–4, is one of the characteristics of an exchange mechanism. The Coulomb forces improve the results by a percentage of about 16.2589%, which is not small and so Coulomb forces are very important and must be included in the three-body calculations.

Thus, we can conclude that the present three-body treatment of direct reaction mechanism gives the shape of angular distributions, but with more structure than that given by the Born approximation. Also, it takes into account explicitly the nonadiabatic effects of the interaction. Since the equations obtained have the form of Lippmann–Schwinger equations, they are thus an exact optical model. This makes the present model a good theory of direct transfer nuclear reactions, but rather it is an exact theory which has the essential Born approximation of direct transfer nuclear reactions.

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